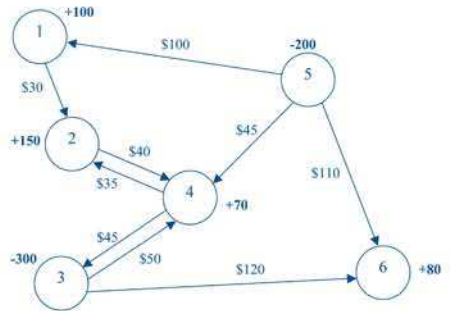
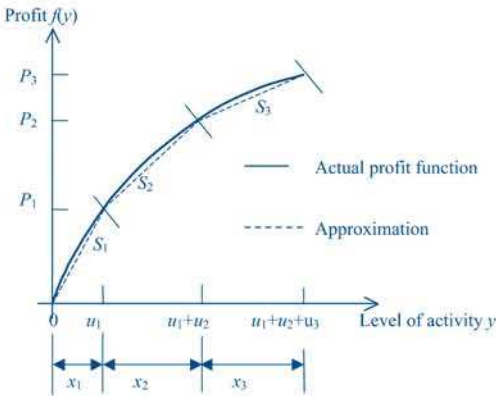
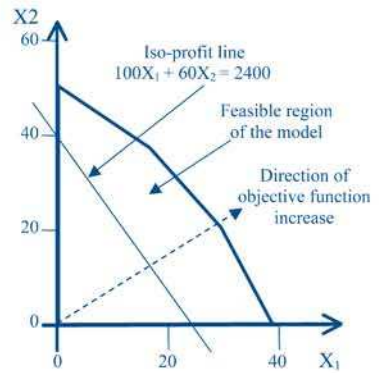


# Optimization Modelling

## A Practical Approach



Ruhul A. Sarker  
Charles S. Newton

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