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# Mathematics for Engineers

Francesc Pozo Montero, Núria Parés Mariné, and Yolanda Vidal Seguí

A Chapman & Hall Book

# Mathematics for Engineers

**Mathematics for Engineers** offers a comprehensive treatment of the core mathematical topics required for a modern engineering degree. The book begins with an introduction to the basics of mathematical reasoning and builds up the level of complexity as it progresses. The approach of the book is to build understanding through engagement, with numerous exercises and illuminating examples throughout the text designed to foster a practical understanding of the topics under discussion.

**Features**

- Replete with examples, exercises, and applications
- Suitable for engineers but also for other students of the quantitative sciences
- Written in an engaging and accessible style while preserving absolute rigor.

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*With gratitude to our students and colleagues, whose curiosity and insights enriched this work, and to our families and friends, whose support and encouragement made this book possible.*

# Contents

# *Preface*

The book you hold in your hands (whether in physical or digital format) is the result of over twenty-five years of teaching mathematics in engineering degrees. The book follows the basic scheme of a core calculus course in one variable of the new degrees adapted to the European Higher Education Area, but includes three additional chapters: one dedicated to mathematical reasoning and proof methods, another dedicated to numerical sets, with special emphasis on complex numbers, and a final chapter intended as an introduction to linear algebra, in which matrices and systems of linear equations are presented.

Throughout these years of teaching and research, we have imparted substantial knowledge but without any doubt, it is these students who have taught us how to approach our teaching to be better teachers! In this sense, this book aims to reflect the way of studying of the students that we can base on the necessary compromise between the quantity of knowledge we want to convey and the quality of that knowledge or how that knowledge is acquired. Formalisms sometimes put obstacles to understanding. But without formalisms, new knowledge cannot be built.

This book reflects a rigorous and precise mathematical approach combined with a clear didactic intention. In each chapter of the book, we wanted to justify everything that is exposed, proving most of the theorems and propositions, but without that generating problems or hindering reading. The basic scheme followed by each of the chapters is the following:

- An introduction, which briefly presents the contents of the chapter.
- The development of theoretical concepts and procedures, with a significant number of examples and demonstrations of theorems.
- A series of Proposed Exercises, interspersed in the text, to check the level of achievement of the material. The solution to the Proposed Exercises is found in an Annex (www.Routledge.com/9781032505442).
- Solved Problems at the end of each chapter, organized according to theme and degree of difficulty.

In particular, and as a demonstration of the effort we have put into transmitting mathematics in the most didactic and clear way possible, the book includes more than 300 figures and more than 200 solved problems.

We sincerely believe that this book can be an excellent tool to guide and complete the study of a calculus course in one variable in a face-to-face study, but it can also be used as basic support material in calculus courses taught at a distance.

We cannot conclude this prologue without thanking those who have helped us, in one way or another, to make this book a reality. Thus, we thank Daniel Amposta Navarro, Ignacio Arruga Cantalapiedra, Sergio Arruga Cantalapiedra, Antonio de la Casa Gómez, and Jesús Martínez Fernández.

Barcelona, August 27, 2024.

Francesc Pozo Montero
Núria Parés Mariné
Yolanda Vidal Seguí

# *Author Biographies*

**Francesc Pozo Montero** received his degree in Mathematics from the University of Barcelona in 2000 and completed his PhD in Applied Mathematics at the Universitat Politècnica de Catalunya (UPC) in 2005. Since 2000, he has been a member of the Department of Mathematics at UPC, where he currently holds the position of Full Professor. He also serves as the coordinator of the Control, Data, and Artificial Intelligence research group and is recognized as a Senior Member of IEEE.

Francesc Pozo Montero's research interests include condition monitoring, control systems, data-driven modeling, system identification, and structural health monitoring, with a particular emphasis on applications related to wind turbines.

He is an Editorial Board Member for several prestigious international journals, including *Structural Control and Health Monitoring* (Wiley), *International Journal of Distributed Sensor Networks* (Wiley), *Mathematical Problems in Engineering* (Wiley), *Mathematics* (MDPI), *Sensors* (MDPI), *Algorithms* (MDPI), *Journal of Vibration and Control* (Sage), *Frontiers in Built Environment* (Frontiers), *Frontiers in Energy Research* (Frontiers), and *Energies* (MDPI).

Francesc Pozo Montero has made substantial contributions to his field, authoring over 70 high-impact journal articles, participating in 23 competitive R&D&I projects, and writing 34 book chapters and 12 books. He has successfully supervised six PhD candidates, filed one invention patent, and established a collaboration contract with an industry partner. His work has also been presented in more than 130 conference papers.

**Núria Parés Mariné** holds a degree in Mathematics (1999) and a PhD in Applied Mathematics (2005) from the Universitat Politècnica de Catalunya (UPC), Barcelona, Spain. As a Full Professor at UPC, she focuses on developing and advancing numerical methods applied to engineering. Over the years, her research has concentrated on three primary areas: result certification, model reduction techniques, and machine learning. Through her innovative approach, she has successfully collaborated with internationally renowned researchers, contributing significantly to the academic and scientific community.

Núria Parés Mariné has made substantial contributions to her field, authoring over 25 high-impact journal articles and participating in 19 competitive R&D&I projects. Her prolific output also includes writing three book chapters and ten books. She has guided and supervised three PhD candidates to successful completion, showcasing her dedication to mentoring the next generation of researchers. In addition, she has been involved in technology transfer, establishing a collaboration contract with an industry partner and bridging the gap between academia and real-world applications.

Her work has been widely recognized and presented in more than 65 conference papers, illustrating her commitment to disseminating knowledge and sharing advancements with the broader scientific community. Prof. Parés continues to be a leading figure in her research areas, driving innovation and excellence in applied mathematics. Her ongoing efforts in integrating machine learning with traditional numerical methods are paving the way for new, groundbreaking approaches in engineering and related disciplines.

**Yolanda Vidal Seguí** holds a degree in Mathematics (1999) and a PhD in Applied Mathematics (2005) from the Universitat Politècnica de Catalunya (UPC), Barcelona, Spain. As an Associate Professor at UPC and an IEEE Senior Member, she is deeply involved in multidisciplinary research, with a particular focus on the application of her expertise to wind turbines. Her research areas encompass condition monitoring, structural health monitoring, fault diagnosis and prognosis, predictive maintenance, machine learning, deep learning, artificial intelligence, and mathematical modeling.

Yolanda Vidal Seguí serves on the Editorial Board of several prestigious international journals, including Engineering Applications of AI (Elsevier), Wind Energy (Wiley), Wind Energy Science (Copernicus), Journal of Vibration and Control (SAGE), IET Renewable Power Generation (IET), Mathematics (MDPI), Sensors (MDPI), Energies (MDPI), Frontiers in Built Environment, and Frontiers in Energy Research.

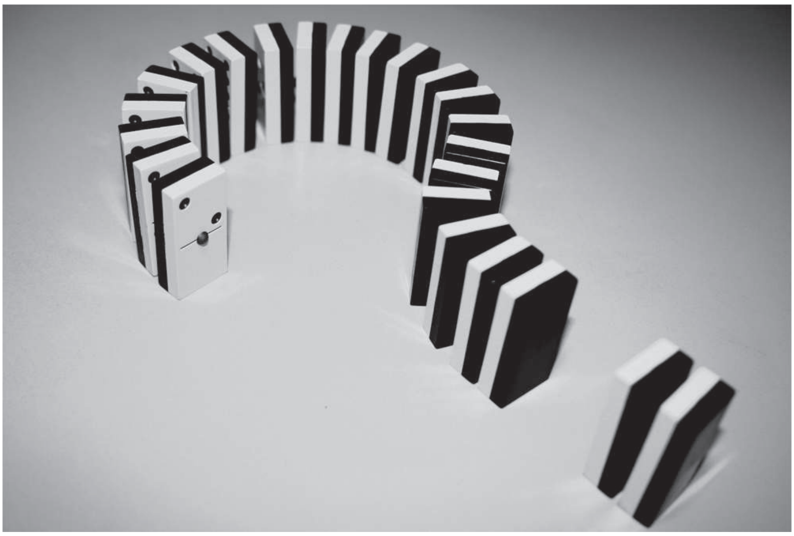
Her prolific contributions to the field are demonstrated by over 70 high-impact journal articles, 23 competitive R+D+I projects, 17 book chapters, and 10 books. She has supervised 7 PhD theses, with 4 currently ongoing. Additionally, she holds 1 invention patent and has secured a collaboration contract with an industry partner. Dr. Vidal has also presented 120 conference papers, further showcasing her dedication and impact in her areas of expertise.

# Part I

# Mathematical Reasoning

# 1

# *Introduction to Mathematical Reasoning*



*«If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.»* (John von Neumann)

In this first introductory chapter, the different mechanisms that mathematics uses to prove various types of properties are presented. Among the different strategies presented (direct proof, contrapositive, or proof by contradiction), mathematical induction stands out, which allows proving properties about natural numbers.

## 1.1 Mathematical Reasoning

According to the 22nd edition of the Spanish Language Dictionary, mathematics can be defined as follows:

**Definition 1.1** (Mathematics)**.** Deductive science that studies the properties of abstract entities, such as numbers, geometric figures, symbols, and their relationships.

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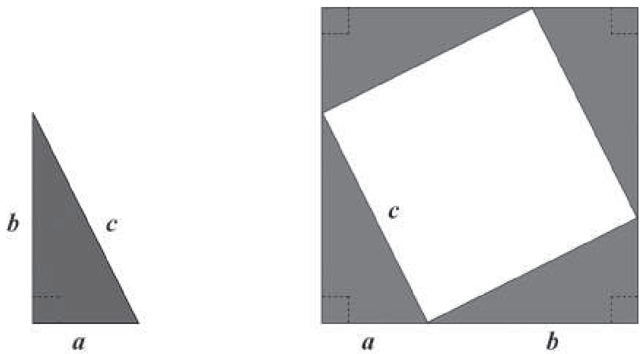


**FIGURE 1.1**
In a right triangle with hypotenuse $c$ and legs $a$ and $b$, we have $a^2 + b^2 = c^2$.

For example, the Pythagorean theorem (Figure 1.1) tells us that if we have a right triangle with sides $a$ and $b$ and hypotenuse $c$, then the sum of the squares of the legs is equal to the square of the hypotenuse, that is:

$$a^2 + b^2 = c^2.$$

In this chapter, we will see some of the tools that mathematics uses to prove these types of properties.

Mathematical properties or statements are called different names depending on their importance. The most important properties are called **theorems** (such as the Pythagorean theorem), while lemmas, corollaries, or propositions collect less important properties. Let's see some more formal definitions.

**Definition 1.2** (Theorem)**.** A proposition that can be logically proven from axioms or other theorems already proven, using accepted rules of inference.

**Definition 1.3** (Lemma)**.** A proposition that must be proven before establishing a theorem.

**Definition 1.4** (Corollary)**.** A proposition that does not require a particular proof but is easily deduced from what has been proven before.

**Definition 1.5** (Proposition)**.** A statement of a truth that has been proven or that is being tried to be proven.

The importance of theorems, lemmas, corollaries, or propositions is that they collect properties that **have been proven to be true** (following the logical-deductive method characteristic of mathematics).

But what is a mathematical proof?

**Definition 1.6** (Proof)**.** A mathematical proof is a **coherent sequence of steps** that allows one to ensure the truth of a statement.

The starting point of a proof is a set of statements that are considered true and are called **premises**. These premises can be hypotheses, axioms, or other propositions or theorems previously proven. Considering the premises as true, one must arrive at the original statement by **applying certain logical rules or applying previously proven properties**.

## 1.2 Is It Easy to Make a Mathematical Proof?

In general, proving a mathematical property can be very complex. The key questions we must ask ourselves are:

- Where do I start?
- What steps should I follow?

That is, the difficulty lies in knowing how to choose the premises well and then knowing which logical reasoning to follow to arrive at the original statement.

In this book, we will present and review basic concepts of calculus in one variable and algebra, delving into reasoning and proofs.

## 1.3 Methods of Proof

Although there is generally no single procedure for proving theorems, there are different **methods of proof** that are commonly used in mathematics. The five basic types of reasoning are:

- Direct proof.
- Proof by contrapositive.
- Proof by contradiction or reductio ad absurdum.
- Proof by counterexample.
- Proof by induction.

Next, we will see several examples of simple proofs obtained from these four methods.

Suppose we want to prove that a statement is true. We will call this statement $Q$. For example, let's imagine that we want to prove that if $a$ and $b$ are two real numbers, then the square of their sum is equal to the square of the first term plus the square of the second term plus twice the first term times the second term, that is:

$$(a+b)^2 = a^2 + b^2 + 2ab.$$

To prove $Q$, we generally need to start with some premises and logical rules. We will call these properties $P$. In the case of wanting to prove that $(a+b)^2 = a^2+b^2+2ab$, the premises