Artyom M. Grigoryan • Sos S. Agaian

Quantum Image Processing in Practice

A Mathematical Toolbox

with website

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To Anoush
and
To Sarkis and Gayane

Contents

Preface

The modern world has witnessed remarkable applications in the dynamic field of image processing, where operations transform an image to enhance it or extract vital information. It is a vibrant and diverse field encompassing various applications, such as facial recognition, image segmentation and compression, noise reduction, and more. These applications require sophisticated techniques to transform, enhance, and extract image information. However, these techniques also demand substantial computational resources for image storage and processing, which pose significant challenges for scalability and efficiency. Therefore, there is a critical need for more advanced and innovative methods to handle visual information. On the other hand, quantum computing defines a probabilistic approach to represent classical information using methods from quantum theory. Quantum computing offers a probabilistic and parallel approach to computation, which differs fundamentally from the deterministic and sequential approach of classical computing. The basic unit of quantum information, the qubit, can exist in a superposition of two states until measured, which enables quantum parallelism and entanglement. These quantum phenomena can provide exponential speedups and enhanced security for specific computational tasks, such as factoring large numbers, searching unsorted databases, simulating quantum systems, and solving linear systems of equations.

Quantum image processing (QIP) is a research branch of quantum information and computing that aims to exploit the advantages of quantum computing for image processing. QIP studies how to encode and process images using various quantum image representations and operations in a quantum computer. QIP has the potential to outperform classical image processing in terms of computing speed, security, and minimum storage requirements. However, QIP also faces many challenges and open questions, such as quantum superiority, reading the classical data, measurement, noise and error correction, scalability and compatibility, and the practical implementation of QIP algorithms and circuits.

In this book, we provide a comprehensive introduction to QIP, covering the theoretical foundations, methodological developments, quaternion color imaging, and practical QIP applications. We describe the existing quantum image representations and their operations, such as geometric transformations, color transformations, filtering, and enhancement. We also explore the emerging topics and applications of QIP, such as quantum image filtration in the frequency domain, convolution, and fast unitary transforms. We discuss the current state of QIP research, addressing the controversies and opportunities, as well as the challenges and future directions of QIP. We illustrate the QIP algorithms and circuits with detailed examples, diagrams, and code snippets using the Qiskit framework. We also provide exercises and references for further learning and research.

Organization of the Book

This book is organized into 14 chapters, as follows:

Chapter 1: *Introduction*
This chapter provides an overview of the main concepts and motivations of quantum computing and image processing. It outlines the structure and objectives of the book.

Chapter 2: *Basic Concepts of Qubits*
This chapter delves into the core concepts and principles, such as computational qubit states, superposition, operations on qubits, permutations, elementary gates, and qubit measurement. It presents the operations and gates in matrix and graphical notations and illustrates them with examples; 3-D model of qubits is presented together with the known Bloch sphere.

Chapter 3: *Understanding 2-Qubit Systems*
This chapter focuses on 2-qubit systems, which are the building blocks of multi-qubit systems. It discusses the mathematical tools and techniques for manipulating 2-qubit systems, such as projection operators, Kronecker product and sum, qubit entanglement, orthogonality, and unitary transformations. It also describes the elementary operations and main gates for 2-qubit systems, such as CNOT, SWAP, local and controlled gates, and explores their properties and applications with practical examples.

Chapter 4: *Multi-Qubit Superpositions and Operations*
This chapter extends the concepts and methods of 2-qubit systems to multi-qubit systems, which are essential for quantum image processing. It examines multi-qubit superpositions of different types. Many 3-qubit gates with 1 and 2 control bits are described with matrices and circuit elements. It also highlights the key 3-qubit gates, such as Toffoli and Fredkin, bit SWAP, and Hadamard gates, and shows how they can be used to implement classical logic functions and reversible circuits.

Chapter 5: *Fast Transforms in Quantum Computation*
This chapter introduces the quantum analogs of the classical fast transforms, such as the discrete paired, Fourier, and Hadamard transforms which are widely used in image processing. It provides detailed descriptions of the algorithms and implementations of these quantum-fast transforms, supported by examples and circuit designs. It also compares the advantages and disadvantages of these quantum fast transforms concerning their classical counterparts. Examples and circuits of these transforms and their inverses on 2-, 3-, and 4-qubits are presented. The paired transform is the core of the Fourier and Hadamard transforms. Therefore, the quantum paired transform is described in detail. The 1-D quantum circular convolution for phase filters with circuits is also presented with examples.

Chapter 6: *Quantum Signal-Induced Heap Transform*
This chapter presents a novel concept of quantum fast transform, which refers to the so-called discrete signal-induced heap transform (DsiHT), which can generate a unique unitary and fast transform for any given signal. It explains the theory and algorithm of quantum signal-induced heap transform (QsiHT) and demonstrates its applications in quantum cosine and Hartley transforms with quantum circuits. It also shows how DsiHT can be used to factorize and decompose any transform in a set of rotated gates and permutation, as well to initiate any quantum superposition from the basis state 0.

Chapter 7: *Quantum Image Representation with Examples*
This chapter describes the various quantum image representations proposed in the literature and compares their features and limitations. It covers the models for both grayscale and color images, such as QLM, NEQR, FRQI, RKL, GQIP, QIRHSI, and MQFTR. It also presents the 2×2 model of color image respresentation as a single

grayscale image in quantum computation. It explores the quantum representation of different color models, such as RGB, CMY, XYZ, HSV, and HSI, and discusses the challenges and opportunities of quantum color image processing.

Chapter 8: *Image Representation on the Unit Circle and MQFTR*
This chapter focuses on a specific type of quantum image representation, called the multi-qubit Fourier transform representation (MQFTR), which encodes the image information on the unit circle using the Kronecker product of qubits. It explains the advantages and disadvantages of MQFTR. It presents some extensions and variations of MQFTR, such as MQFTR with phase shift and MQFTR with amplitude modulation. It also describes the quantum schemes for 2-D quantum Fourier transform with examples for 4×4 and 8×8 images.

Chapter 9: *New Arithmetic on Qubits*
This chapter introduces some novel concepts and methods of arithmetic operations on qubits, such as multiplication, division, conjugate, and inverse. It extends these operations to multi-qubit superpositions and discusses their properties and applications. It also shows how these operations can be implemented using quantum circuits and algorithms in image summation, linear convolution and filtration with examples.

Chapter 10: *Quaternion-Based Arithmetic in Quantum Image Processing*
This chapter explores the non-commutative quaternion arithmetic in quantum color image processing, which can offer some advantages over conventional complex arithmetic. It differentiates between the traditional and commutative quaternion algebras and discusses their properties and applications. It presents a new concept of the multiplicative group of 2-qubits and describes the main properties of the multiplication of 2-qubits and 2-qubit-based superpositions. It also describes the graphical representation of 2-qubits.

Chapter 11: *Quantum Schemes for Multiplication of 2-Qubits*
This chapter presents detailed quantum 2-qubit multiplication circuits that underlie many quantum arithmetic operations. It showcases a few circuits composed using the QsiHT concept and compares their performance and complexity. It shows how to design the quantum 4×4-gates of multiplication with 4, 5, and 6 rotations. It also describes 12 Hadamard matrices as multiplication gates.

Chapter 12: *Quaternion Qubit Image Representation (QQIR)*
This chapter introduces a new quantum image representation, called the quaternion qubit image representation (QQIR), which combines the features of 4-D quaternion arithmetic and MQFTR. It covers some basic operations in QQIR, such as square root, power, and exponentiation, and shows how they can be used for image processing. It explores some advanced operations in QQIR, such as the convolution and gradient calculation, and demonstrates their applications in image filtering, edge detection, and feature extraction. It also describes the concept of the quantum quaternion Fourier transform (QQFT) and ideal filtration by this transform.

Chapter 13: *Quantum Neural Networks (QNN)*
This chapter bridges quantum computing and machine learning and discusses the development and applications of quantum neural networks inspired by classical neural networks. It highlights the differences, synergies, and challenges of quantum and classical neural networks and reviews some existing models and architectures of quantum neural networks. It also explores some potential applications of quantum neural networks in image processing, such as image classification, recognition, segmentation, restoration, and reconstruction.

Chapter 14: *Conclusion and Opportunities and Challenges of Quantum Image Processing*
This chapter summarizes the main contributions and findings of the book and reflects on the current state and future directions of quantum image processing. It discusses the opportunities and challenges of quantum image processing, such as quantum superiority, noise, and error correction, scalability and compatibility, and practical implementation. It also provides some suggestions and recommendations for further research and development in quantum image processing.

Designed for a diverse audience, from students to professionals, this book is accessible to those with some background in linear algebra, quantum mechanics, and image processing. Many examples and references throughout the text encourage further exploration and deeper understanding. We are grateful to everyone who read this book. We hope the reader will enjoy learning about quantum imaging and its applications and gain some ideas and inspiration from the methodologies and concepts discussed in this book. This book explores the emerging interdisciplinary field of quantum imaging, introducing fundamental concepts, state-of-the-art techniques, and applications.

An Invitation to Innovation: We also hope that this book will arouse readers' interest in QIP and inspire them to contribute to its development. Join us on a journey where the classical and quantum worlds intertwine, unlocking a future brimming with unprecedented potential for image processing innovation.

We appreciate all who assisted in the preparation of this book. We are grateful to Meruzhan Grigoryan, Alexis Gomez, and the reviewers for many suggestions and recommendations.

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Finally, we are grateful for the opportunity to present "*Quantum Image Processing: A Mathematical Toolbox*," a work that has been a labor of love and dedication.

About the Companion Website

This book is accompanied by a companion website

www.wiley.com/go/grigoryan/quantumimageprocessing

This website includes:

- QuAlgorithms

Part I

Mathematical Foundation of Quantum Computation

1

Introduction

Image processing represents a critical use of artificial intelligence in various applications, including biomedicine, entertainment, economics, and industry. For example, image processing is extensively used in fast-growing markets like facial recognition and autonomous vehicles. Recently, the rapidly increased volume of image data has become the critical driving force for further improving image processing and analysis efficiency. Quantum computing offers great promise for speedy computation of problems in digital image processing (DIP), namely, in processing grayscale and color images. Quantum image processing (QIP) is a research branch of quantum information and quantum computing. It studies how to use quantum mechanics' properties to represent images in a quantum computer and then implement various image operations based on that image format. The parallel execution of multiple computations is a quantum computer's main advantage [1, 2], which can be used for many image processing applications. Therefore, developing new tools for calculating not only known procedures in DIP but also new ones is essential, using special rules for calculating qubits. The basic unit in quantum computation is a single qubit with only two states. The mathematical description of a qubit is probabilistic. The qubit can be measured only once, and at any moment of its measurement, it will be in only one of its states 0 or 1. If we consider qubits as spins, then these two states are "spin up" and "spin down." Before the measurement, the state of the qubit is described as a linear combination of the basis states 0 and 1. The amplitudes of such a combination are defined by probabilities of the qubit being in these states. Namely, these amplitudes are square roots of the corresponding probabilities. Thus, in quantum computation, there are no numbers greater than 1, such as 2, 3, 5, and 12. Such numbers can only be written in basis states of multi-qubit superpositions. It is possible only to work and get such state numbers through their amplitudes. In other words, through the probabilities of qubits to be in these states. This is the main difference between the existing classical and future quantum computers. For example, to add two 3-bit numbers 3 and 4 and get 7 is a trivial operation for the classical computer. Solving this equation $3 + 4 = 7$ in a quantum computer with simple 3-bit operations $\oplus$ is impossible. All existent algorithms for the addition of numbers use more bits, or qubits, and other operations. In general, the composition of algorithms for processing images is a complex task. Storage and processing of information about 2^r states in one superposition of r-qubits is the main factor and a big advantage in quantum computation (QC). Therefore, there is an opinion that a quantum computer may surpass the much real possibilities of existing computers in the near future. All this impeded the development of image representation, storage, and processing methods in QC. In quantum imaging, several methods are currently available, such as interference, correlation, and entanglement-based quantum mapping [3–5]. These are the known methods in use with applications in quantum computation. Unlike the classical computer, images in QC can be represented by different quantum superpositions of states. These models include the qubit lattice model (QLM) [6], flexible representation for quantum images (FRQI) [7], the novel enhanced quantum representation (NEQR) [8–10], novel representations of color images [11, 12], new models of the Fourier transform quantum representation (FTQR) [12, 13], and quaternion qubit image representation (QQIR) [14, 15].

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This book aims to introduce the reader to the necessary part of QIP that plays an essential role in quantum information processing. It includes

- A review of some needed mathematical concepts of qubits and their operations, such as multiplication, division, inverse qubits, and their quantum circuits description.
- the quantum Fourier and Hadamard transform with their quantum circuits description.
- the processing images in the frequency domain, namely, applying quantum ideal low-pass and high-pass filters.
- the quantum grayscale and color image quaternion-based representations that allow efficient encoding of the classical data into a quantum state and future use in QIP applications.
- the basics of quantum neural networks.

The study shows that the number of researchers working in the QIP field is increasing, and some significant problems remain unsolved. We hope this book will accelerate the efforts to create more practical QIP-based technologies.

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2

Basic Concepts of Qubits

In this section, the basic concept and operations, called gates, on qubits are described. We are interested in the mathematical side of this concept, namely the mathematical model of the qubit. The main gates are considered and then, in Section 9.1, we present our vision of a multiplicative qubit group using qubit multiplication and division operations. One can even solve quadratic equations with qubits.

In the theory of quantum computation, a quantum bit, which is called a qubit, may be in two states, $|0\rangle$ and $|1\rangle$. We can think of a qubit as a magical, fast-spinning 2D coin with two numbers 0 and 1 written on its sides. When we stop the rotation with our hand, we will see only one side of the coin in the palm of our hand, that is, only the number 0 or 1 (see Fig. 2.1). The events that the coin will show 0 or 1 may occur with different probabilities p_1 and p_2. The square roots of these probabilities, $a_0 = \sqrt{p_0}$ and $a_1 = \sqrt{p_1}$, are called the amplitudes of the qubit. These two events are called the basis states of the qubit. The state 0 is usually written with 2 bits as 10, and the state 1 as 01. Mathematically, a single qubit is defined as a linear combination of these basis states with given amplitudes a_0 and a_1. Any quantum system with only two states can be considered as a qubit. For example, (i) a nuclear spin with two energy levels and (ii) a photon with vertical and horizontal polarization.

For column and row vectors, Dirac's ket-bra notation is used. Ket-notation $|\boldsymbol{x}\rangle$ is the column vector for $\boldsymbol{x}$, and bra-notation $\langle\boldsymbol{y}|$ is the row vector for $\boldsymbol{y}$. The state 0 as the unit vector $[1, 0]'$ is the ket-0$|0\rangle$, and the basis state 1 as the vector $[0, 1]'$ is the ket-1 $|1\rangle$. Here, the symbol$'$ is used for the transpose vector. The operation $\langle\boldsymbol{y}|\boldsymbol{x}\rangle$ is the inner product, that is, $\langle\boldsymbol{y}||\boldsymbol{x}\rangle$. For examples, if $\boldsymbol{x} = [1, 2]$ and $\boldsymbol{y} = [3, -1]$, then

$$\langle\boldsymbol{y}|\boldsymbol{x}\rangle = \langle\boldsymbol{y}||\boldsymbol{x}\rangle = \boldsymbol{y}\boldsymbol{x}' = 1\cdot 3 + 2\cdot(-1) = 1.$$

If the vectors are complex, the inner product is defined as $\langle\boldsymbol{y}|\boldsymbol{x}\rangle = \langle\boldsymbol{y}|\overline{|\boldsymbol{x}\rangle}$. The bra vector $\langle\boldsymbol{x}|$ is complex conjugate of $|\boldsymbol{x}\rangle$.

In a real Euclidean space, the inner product is defined as a rule by means of which for each of the two elements x and y of the space a real number is associated (*inner product*), that is denoted as (x, y) and that satisfies the following four Axioms:

1) $(x, y) = (y, x)$;
2) $(x + z, y) = (x, y) + (z, y)$;
3) $(kx, y) = k(x, y)$, $k \in R$;
4) $(x, x) = |x|^2 \geq 0$, and $(x, x) = 0$ if only $x = 0$.

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