

Machine Learning *with* Python

PRINCIPLES AND PRACTICAL
TECHNIQUES

Parteek Bhatia



Machine Learning with Python

Machine learning (ML) has become a dominant problem-solving technique in the modern world, with applications ranging from search engines and social media to self-driving cars and artificial intelligence (AI). This lucid textbook is an effort to present the contents in a simplified and practical manner without losing the details of the subject.

The book presents the theoretical foundations of ML algorithms, and then illustrates each concept with its detailed implementation in Python to allow beginners to effectively implement the principles in real-world applications. All major techniques, such as regression, classification, clustering, deep learning, and association mining, have been illustrated using step-by-step coding instructions to help inculcate a “learning by doing” approach. The book has no prerequisites, and covers the subject from the ground up, including a detailed introductory chapter on the Python language. As such, it is going to be a valuable resource not only for students of computer science, but also for anyone looking for a foundation in the subject, as well as professionals looking for a ready reckoner.

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Principles and Practical Techniques

Parteek Bhatia



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